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STATISTICAL ANALYSIS OF ENERGY-TRANSFER PROCESSES IN MAGNETOMOVABLE ARC

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A successive selection of the dominant variables in a generalized volt-ampere characteristic of a magnetomovable arc is performed by regression-analysis methods. It is shown that turbulent energy transfer increases in importance with the electrode gap and magnetic induction.

1. Introduction. An electric-arc discharge is a complex phenomenon, in which numerous interrelated processes of different nature occur. Unstable intricate-configuration discharges practically defy mathematical modeling and we have to rely on experimental data. The application of the theory of similarity, in this case, makes obtaining the necessary information much easier. However, in the experiment, only a few variables can be varied, on which many processes in the discharge depend. The number of possible generalized arguments is governed by the number of variables, therefore the similarity proves to be approximate. Accordingly the problem arises of selecting the most important processes that have a major effect on discharge characteristics. It is complicated further by the fact that the character of the dominant processes depend on discharge conditions.

If the dependence of the dominant mechanisms on discharge conditions proves to be sufficiently strong, when deriving expressions for discharge characteristics we have to seek a compromise between the generality and accuracy of the formulas derived. But, to solve this problem, we need, first, to have data on the degree of the influence of changing conditions on the character of the dominant processes.

An electric arc is an object in which electric energy is intensely converted to heat energy and the latter is removed from the discharge by different mechanisms of heat transfer. Therefore, the characteristics of the arc discharge depend primarily on heat and mass-transfer processes. A convincing example of the strong influence of the conditions of arc burning on the mechanism of energy transfer is an arc that moves along the electrodes under the action of a magnetic field (Fig. 1). This type of discharge is very unstable, and in it, turbulent heat transfer can have a sufficiently strong effect along with other modes of energy transfer.

We investigated the influence of the magnitude of an electrode gap and magnetic induction on the character of energy transfer. For this purpose, we selected successively the most important similarity numbers for an arc that moves in air under the action of an external magnetic field along parallel electrodes ("railtron") and rotates between annular electrodes ("coaxial plasmatron".) We tested the similarity numbers for different mechanisms of energy transfer: $\pi_{conv} = \rho_0\sigma_0^2h_0^2BL^5/I^3$ (convection); $\pi_{cond} = \sigma_0\lambda_0T_0L^2/I^2$ (conduction); $\pi_{rad} = \sigma_0Q_{rad.0}L^4/I^2$ (radiation); $\pi_{turb} = \rho_0\sigma_0h_0^{1.5}L^3/I^2$ (thermal turbulence). The influence of gasdynamic processes was allowed for by the Reynolds number, which, in this case, takes the form $Re = (IBL\rho_0/\eta_0^2)^{0.5}$. We also took into account the magnetic induction $\pi_{ind} = \sigma_0^2L^3B^3/\rho_0I$ and the Hall effect $\pi_{Hall} = \sigma_0B/ln_{e0}$.

The volt-ampere characteristic of an electric discharge (VAC) is the most important. In this work, we consider a generalized VAC in which the generalized resistance $UL\sigma_0/I$ is a dependent variable while the above similarity numbers are tested as independent variables. The characteristic is approximated by a power expression. The experimental data on the VAC of the railton are taken from [3], and on the coaxial plasmatron, from [4]. The characteristic values of plasma properties are borrowed from [5].

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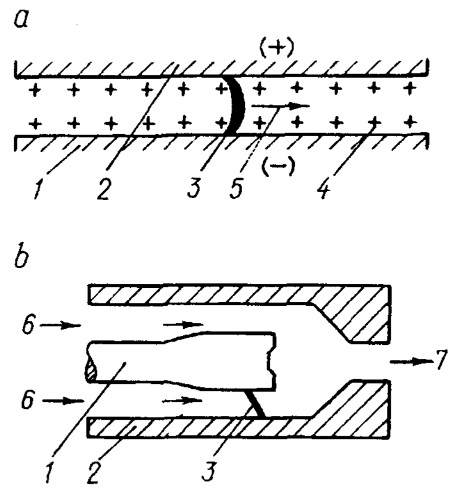

Fig. 1. Diagrams of devices with an electric arc that moves along electrodes in interaction with a magnetic field: a) the arc moves along straight parallel electrodes ("railtron"); 6) the arc rotates in an annular electrode gap ("coaxial plasmatron".) 1, 2) electrodes; 3) arc; 4) magnetic field lines of force; 5) direction of arc column motion; 6) gas supply; 7) plasma outlet.

2. Selection Procedures for the Dominant Similarity Numbers. We sorted the similarity numbers by the degree of their influence on the dependent variable by successive F-tests with standard programs for step-by-step selection from a set of independent variables of the variable that ensures the maximum increase for the given step in the ratio of the regression variance to the mean-square deviation of the residuals from regression $F_{\text{regr}} = S^2_{\text{regr}}/S^2_{\text{rem}}$ [1, 2]. The independent variables were successively selected for individual pairs of values of the gap and magnetic induction. In the experimental data for the railtron used [3], there are 4 pairs of gap magnitudes: 1) (12.7–19.1) mm, 2) (19.1–25.4), 3) (25.4–32.0), and 4) (32.0–38.0) and 4 pairs of magnetic inductions: 1) (0.012–0.025)T, 2) (0.025–0.054), 3) (0.054–0.080), and 4) (0.080–0.108). The discharge occurred in air; the current varied within I = 100–1000 A. In the device with annular [4] electrodes, the discharge also occurred in air; the current varied within I = 100–900 A. In this case, there are also 4 pairs of magnetic inductions: 1) (0.085–0.12)T, 2) (0.12–0.17), 3) (0.17–0.23), and 4) (0.23–0.28) but the gaps had only two values: L = 3 mm and L = 6 mm. Therefore, for the annular electrodes, we investigated only the influence of magnetic induction.

For generalized VACs that included the most important independent variables we also calculated the determination coefficient $R-SQ$, the matrix of the coefficients of the correlation between individual variables, the standard error SE, the Student quantiles t for the coefficients, the Fisher ratio F_{regr} both for the entire multiple regression and its distributions by individual variables that show their relative degree of influence. These quantities were determined for each pair of gaps and magnetic inductions as well as for the entire data array. We also analyzed the residuals for normality, independence, homogeneity, and the strong influence of individual points.

The magnetomovable arc is an unstable object, in which substantial parameter fluctuations are observed. These discharges cannot operate on ascending segments of the VAC, since their transverse dimensions are not limited by the external action. Therefore, a high-current discharge that moves in a magnetic field has a VAC approximately parallel to the axis of the currents $U \approx$ const; the sum of the exponents for the current must be equal to zero. For the generalized VACs in a power approximation this condition leads to the relation $-\sum_i \alpha_i p_i \approx 1$ [5], where α_i is the exponent for the "natural" generalized argument i and p_i is the exponent for the current in this argument. Therefore, along with partial values of the exponents for "standardized" variables and F_{regr}, the value of $\alpha_i p_i$ also characterizes the relative role of the independent variable i.

3. Discussion of the Results. The results of selection of the dominant independent variables for different electrode distances (gaps) of the railtron are given in Tables 1 and 2.

The minus sign on π_{ind} (Table 2) shows that the induced electromotive force acts against the external electric field. At the same time, negative coefficients on π_{conv}, π_{rad}, and Re contradict the physical essence of the

TABLE 1. Particular Values of F_{regr} for Generalized VAC of Railtron Obtained by Step Selection for Different Pairs of Gaps

Gaps	Number of experimental points	Particular values of F_{regr}						
		π_{conv}	π_{cond}	π_{rad}	π_{turb}	Re	π_{ind}	π_{Hall}
1	76	2182	–	–	–	30.83	8.510	–
2	66	–	–	–	10,000	–	–	411.4
3	65	11.31	–	–	46.01	–	–	21.26
4	76	–	–	36.25	978.5	–	–	50.28

Note: Dashes in Tables 1-4 show that this number is not included in the model in selection. From Table 1, it can be seen that conduction has no effect on either pair of gaps in the railtron.

TABLE 2. Values of Coefficients of Regression Equation, Multiple Correlation Coefficient, and Standard Error for Generalized VAC of Railtron Obtained by Step Selection for Different Pairs of Gaps

Gaps	$R-SQ$	SE	Coefficients of regression equation						
			π_{cond}	π_{rad}	π_{turb}	Re	π_{ind}	π_{Hall}	$\sum_i \alpha_i p_i$
1	0.995	0.0508	0.344	–	–	−0.09	−0.02	–	1.102
2	0.995	0.0546	–	–	0.558	–	–	0.164	1.116
3	0.996	0.0495	−0.370	–	1.127	–	–	0.506	1.144
4	0.998	0.0485	–	−0.108	0.664	–	–	0.252	1.112

process. These variables are included in the model by the selection program only formally since they ensure the maximum reduction of error.

With allowance made for the indicated comments it turns out that a substantial difference between the first pair of the gaps (small electrode distances) and the remaining pairs is observed. For small values of L convective heat transfer dominates solely while as the distance between the electrodes increases, turbulence begins to play a leading part and the Hall effect has some impact. The turbulent energy transfer increasing in importance with the electrode gap is due to the increase in discharge instability. A stabilizing factor for the magnetomovable arc is the jets of evaporating electrode material but their influence becomes weaker as L increases. In the largest gap, however, an instability decay is observed. Of interest is the pronounced influence of the Hall effect at atmospheric pressure and not very large values of the magnetic induction.

Another interesting factor is elimination of the Reynolds number from the important variables for large gaps. The Reynolds number is the main factor that reflects the influence of gasdynamics on heat-transfer processes. As it increases, energy transfer by gasdynamic turbulence, which increases with the gap, enhances. But the analysis shows that the increase in instability leads to the enhancement of heat transfer by thermal turbulence rather than by gasdynamic turbulence. This fact is explained by the relative value of the thermal and kinetic energies of the flow in subsonic plasma flow. For example, for air with a temperature that is characteristic of arc discharge ($T = 10^4$ K) and Mach number $M = 1$, the plasma enthalpy is by an order of magnitude higher than the specific kinetic energy of the flow. For subsonic velocities, this relation can be several orders of magnitude. Therefore, the primary gasdynamic disturbance of the plasma causes temperature fluctuations and temperature turbulence, whose influence on the processes of energy transfer proves to be much stronger than that of gasdynamic turbulence.

The influence of magnetic induction is shown in Tables 3 and 4. Since all the gaps are included in the analyzed data array for the railtron, the prevailing influence of large electrode gaps led to the elimination from the

TABLE 3. Particular Values of F_{regr} for Generalized VAC of Railtron Obtained by Step Selection for Different Pairs of Magnetic Induction

Gaps	Number of experimental points	Particular values of F_{regr}						
		π_{conv}	π_{cond}	π_{rad}	π_{turb}	Re	π_{ind}	π_{Hall}
1	77	–	–	–	2220	–	15.68	34.41
2	61	–	–	–	4131	–	20.11	78.30
3	58	–	–	12.83	1110	–	–	124.9
4	80	–	–	36.25	978.5	–	–	50.28

TABLE 4. Particular Values of F_{regr} for Generalized VACs of Arc That Rotates Between Annular Electrodes Obtained by Step Selection for Different Pairs of Magnetic Induction

Pairs of magnetic induction	Number of experimental points	Particular values of F_{regr}						
		π_{conv}	π_{cond}	π_{rad}	π_{turb}	Re	π_{ind}	π_{Hall}
1	78	10,000	–	–	–	–	74.0	–
2	52	59.62	5.845	–	5.377	–	–	–
3	65	–	–	–	10,000	4.172	–	52.74
4	91	–	10,000	–	–	933	–	–

model of π_{conv} and π_{ind}, which manifested themselves only for small gaps. The effect of π_{cond} is eliminated as it is in the previous case. Because of inconsistency with physical regularities (the negative coefficients on the given term of the regression equation) we should also eliminate the energy transfer by radiation π_{rad}. Table 3 shows that first an increase in induction causes the development of turbulization, but later on we observe instability decay.

Selection of the most important similarity numbers for the entire set of experimental points revealed two dominant independent variables π_{turb} and π_{Hall} with particular values F_{regr} = 10,000 and 980, respectively. The coefficients on the indicated terms of the regression proved to be 0.544 (for π_{turb}) and 0.160 (for π_{Hall}). The sum of the coefficients α with the corresponding "weights" is equal to $0.544 \times 2 + 0.160 \times 0 = 1.080 \approx 1.0$. The determination coefficient $R-SQ$ = 0.99586 and the standard error $SE = 0.0552$. It seems, however, appropriate to divide the entire set of experimental points into two groups with different conditions of energy transfer: $L \leq 19.1$ mm (the first pair of gaps) and $L \geq 19.1$ mm (the remaining 3 pairs of gaps). A gap $L = 19.1$ mm is included in both groups.

In accordance with the above result, the generalized VAC for $L \geq 19.1$ mm is obtained in the form

$$\frac{UL\sigma_0}{I} = 0.429 \left(\frac{\rho_0 \sigma_0 h_0^{1.5} L^3}{I^2} \right)^{0.556} \left(\frac{\sigma_0 B}{e n_{e0}} \right)^{0.152} . \tag{1}$$

For a logarithmic linear regression, $R-SQ = 0.995$; $SF = 0.055206$; F_{regr} = 13,963; F_{regr} (turb) = 27,140 and F_{regr} (Hall) = 785. Accordingly the error for the power approximation is 5.7%.

For small gaps $L \leq 19.1$ mm, it is appropriate to retain only one independent variable though it leads to an increase in the error. The corresponding formula has the form

$$\frac{UL\sigma_0}{I} = 1.518\left(\frac{\rho_0\,\sigma_0^2\,h_0^2L^5B}{I^3}\right)^{0.325} . \tag{2}$$

For a linear regression in logarithmic coordinates, $R-SQ = 0.944$; $SE = 0.146878$; $F_{regr} = 661$. The relative error for the power approximation is 15.8%, which is much worse than for the large gaps.

A reduction in the error for both small and large L provides additional allowance for π_{ind}.

For the arc that rotates between annular electrodes, the experiments were performed for small electrode gaps of 3 and 6 mm. As the analysis of the data for parallel electrodes shows, in this case, turbulization must be attenuated and convective energy transfer must grow in importance. But in the case under consideration, a stronger magnetic field was applied, which must increase the velocity of arc motion and stimulate instability. Therefore, it is appropriate to consider the influence of magnetic induction on the character of energy transfer in the arc column. The corresponding data of the step selection are given in Table 4.

Because of inconsistency with the regularity, Re *in the third induction pair should be eliminated from the* indicated variables.

Table 4 shows a considerable spread of the selected variables in different pairs of magnetic induction. Nonetheless, the assumption of the development of instability as the magnetic field enhances is confirmed (the second and the third induction pair). However, a further increase in magnetic induction leads to a sharp instability decay (the fourth induction pair). It is difficult to explain this phenomenon by ordinary parameter straggling. Apparently, there is a metamorphosis in the configuration of the arc column. Supposedly, the orientation of the discharge plane changes.

The enhancement of the magnetic field is due to an increase in the velocity of arc motion. The discharge column "collapses" under the action of the force of the interaction between the current and the magnetic field and the counterforce of the aerodynamic drag of the cold flow [6]. This leads to an increase in the drag of the column and discharge destabilization. The arc's tendency to a reduction of drag causes the discharge plane to rotate edgewise to the counterflow when the velocity of column motion attains a critical value. This metamorphosis involves variations in the character of heat transfer. Attenuation of the pressure of the cold gaseous counterflow on the arc column causes instability decay and a reduction in turbulent energy transfer. Energy removal by convective flows is hindered, too, due to the lengthening of the path of motion. Before the rotation of the arc, plasma moved within the column under the action of the magnetic field toward the cold flow along the small axis of the column cross-section [6], while after the rotation the motion is oriented along the large axis. In this situation, the main load of removing Joule dissipation energy from the discharge must be taken over by radiation and conductive energy transfer. In an air plasma, conductive energy transfer prevails, which is in agreement with the data of Table 4 for the fourth pair of magnetic-induction values. The heat transfer between the exterior surface of the arc column and the cold gas is by convection, which determined the increase in the importance of the Reynolds number.

A similar phenomenon of column rotation is, apparently, bound to occur when a critical value of the gap is attained. This tendency is noted in the data of Tables 1 and 2.

The dominant influence of conductive heat transfer and convective heat transfer on the limiting values of magnetic induction presupposes the possibility of generalizing the entire data array for a rotating discharge depending on the corresponding criteria. It is precisely those that were selected by the step method. The corresponding formula has the form:

$$\frac{UL\sigma_0}{I} = 5.0\left(\frac{\rho_0\,\sigma_0^2\,h_0^2L^5B}{I^3}\right)^{0.239}\left(\frac{\sigma_0\,\lambda_0T_0I^2}{I^2}\right)^{0.158} . \tag{3}$$

This generalized VAC shows that arc voltage is almost independent of current, since $-\Sigma\, p_i\alpha_i$ = $3 \times 0.239 + 2 \times 0.158 = 1.033$. The determination coefficient for the linear logarithmic regression is $R-SQ$ = 0.996, the standard error is $SE = 0.037$ (the relative error of the power approximation is 3.8%); $F_{regr} = 25{,}573$.

Comparison with the data for the railtron shows that the decrease in instability for small gaps also reduces the data spread by improving approximation accuracy.

4. Conclusions. A transversely-blown electric arc is a very unstable object, and the character of the dominant energy-transfer processes in it depends substantially on the conditions of discharge burning. A change in the electrode gap and induction of the magnetic field causes considerable changes in heat-transfer conditions. For small values of the electrode gap and magnetic induction, convective energy transfer prevails. An increase in these parameters leads first to an increase in discharge instability and the corresponding prevalence of heat transfer by thermal turbulence. However their further increase causes a variation in the configuration of the arc column, attenuation of instability, and the prevalence of conductive energy transfer in the discharge.

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NOTATION

B, magnetic induction; C, constant; e, electron charge; F, Fisher variance ratio; h, enthalpy; I, current; L, characteristic dimension (electrode gap); M, Mach number; n_e, electron concentration; p, exponent for current; Q_{rad}, volume losses by radiation; $R-SQ$, determination coefficient; Re, Reynolds number; SE, standard regression error; S^2, experimental value of variance; T, temperature; t, Student quantile; U, voltage; α, exponent; η, dynamic viscosity; λ, thermal conductivity; π, similarity number; ρ, density; σ, electrical conductivity. Subscripts: ind, induction; conv, convection; rad, radiant; 0, characteristic value; regr, regression; turb, turbulent; Hall, Hall.

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